USING SIMPLE ALGEBRAIC MODELS AND RATING TEST DATA TO PREDICT WATER HEATER ENERGY CONSUMPTION UNDER OFF-TEST CONDITIONS

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ABSTRACT

This paper considers the use of time-integrated algebraic models combined with ratings data to predict annual energy consumption of water heaters for use conditions differing from the conditions used for rating. The approach is exemplified with gas storage tank and tankless water heaters, for which a prescribed *simulated use test* provides published performance ratings. The models are derived by time integration of energy balance equations. Storage tank water heaters are a paradigm, where the key model inputs can be obtained from ratings data. The tankless water heater model derived here proceeds from analyzing a draw during ramp-up and steady-state, treating outlet water that is below setpoint as not useful. Bins in the key use variables (time since last draw and draw duration) are used to collapse the sum over all draws. Similarly simple models that use ratings data are needed for solar and heat pump water heaters.

1. INTRODUCTION

Simple and easy methods that accurately predict the unit-specific annual performance of rated water heaters (WHs) are imminently useful. Performance under various use conditions is needed to optimize the cost-performance tradeoff when choosing between conventional WHs and advanced energy-saving, but more-costly WHs, as is done routinely by manufacturers, distributors, plumbers, salespeople, and consumers. Incentive organizations need to know WH performance across all users in their area if they are going to rationally set incentive levels based upon ensemble performance. Because of the wide range in real use conditions, no single rating can possibly provide good prediction of annual energy usage across the ensemble patterns. This paper provides simple algebraic models to predict annual energy consumption for storage tank water heaters and tankless water heaters, using unit-specific ratings data (with significant shortcomings for tankless).

Models and data are inextricably entwined in these analyses, as in all engineering. For packaged water heaters (WHs), ratings data on individual units are available at (1). By Federal law, all packaged residential WHs must be tested according to a prescribed standard test (2). The rating metrics include the *energy factor* ($EF= Q_{out,test}/Q_{in,test}$). An ideal situation is when the published ratings provide all the inputs needed for calculating unit-specific annual performance. As we shall see, this is mostly true for conventional storage tank water heaters (STWHs), and unfortunately less true for tankless water heaters (TWHs).

The analyses here and WH ratings generally focus on the water heater itself, and exclude the impact of the piping. The distribution system significantly alters the profile out of water heater, to that which the user experiences at the end use spot. Ultimately, there should be some rating developed that includes the distribution system, to correspond better to what the consumer experiences. But to date that has not been done that we know of.

WH models can be classified into two broad classes, *dynamic* and *static* (also called *time-integrated*). Static models here consist of coupled algebraic equations whose solution is gotten without stepping through time. Models are further classified by how the spatial domain is split up. Static models generally have no spatial subdivisions. Model applications depend on the model spatial and temporal resolution. The most useful models for WH analyses are:

Dynamic 3-D models. Three-dimensional finite element models divide the problem's spatial domain into thousands to millions of small sub-domains or nodes. These models are used in designing advanced hardware, but are far too complex and slow for calculating annual energy use.

Dynamic 1-D models. 1-dimensional finite different models divide the spatial domain into a few to 100s of nodes, and apply energy/mass conservation at the nodes at each time step. These models can incorporate stratification and address dynamic issues like runout and sizing (3). A skilled analyst is needed.

Time-integrated energy balance models. These models result from time integration of the governing 1-zone energy balance (4). They predict annual consumption over annual period using one or more algebraic equations, without iteration or time-stepping. Dynamic problems cannot be addressed with static models.

Correlation Models. Correlation-based models are fits of some physically reasonable functional form to time-integrated results of 1-D simulation models over a range of system types and climates, such as (5). These models are used for solar water heater (SWH) performance, but they do not utilize the available annual ratings (6) to increase accuracy and decrease user effort in preparing inputs.

In this paper we demonstrate methods using ratings data with simple algebraic models. The approach is applied to storage tank and tankless water heaters. The standard rating test for residential water heaters is described in Section 2. Gas storage tank water heaters (GSTWHs) are discussed in Section 3. A TWH model is derived from an analysis of a draw event in Section 4. Section 5 provides conclusions and outlines future work.

2. THE STANDARD WATER HEATER RATING TEST

All STWHs in the U.S. below 75,000 Btuh input and all TWHs below 200,000 Btu/hr input must be tested as in (2). The test is a 24-hr simulated use test with six 10.7 gal draws spaced one hour apart starting at t=0, followed by 19 hours of standby. The setpoint is 135 °F. This unrealistic profile is actually just fine for storage tank water heaters, but leads to bias in tankless and heat pump water heater ratings, as in (7). The test is being revised with more realistic draws and lowered setpoints, which will mostly eliminate bias (8). The constants in the algorithms derived here depend on test conditions and will need revision with new test conditions.

WH rating data include values for the energy factor (*EF*), the recovery efficiency (*RE*), the nominal tank volume V_{tank}, and the input power capacity P_{max} (1). Omitting certain corrections not germane here, metrics are defined as (2):

$EF \equiv Q_{out,day}/Q_{in,day}$, where: (1)

$Q_{out,day} = M_{day}c_p\Delta T_{out\text{-}in} = 41{,}092$ Btu/day

$Q_{in,day}$ = measured energy input over the day

$RE_{gas} \equiv Q_{out,draw}/Q_{in,draw}$, where: (2)

$Q_{out,draw} = (M_{draw}c_p\Delta T_{out\text{-}in})$ = energy withdrawn by the first draw of the 24-hour test;

$Q_{in,draw}$ = energy input recovering from that draw.

The recovery efficiency includes both the losses in converting the input fuel to heat in the tank and the losses to the environment during the recovery. The efficiency of converting the input fuel to heat energy in the tank is the *conversion efficiency* η_{conv}. It is slightly larger than RE.

3. GAS STORAGE TANK WATER HEATERS (GSTWH)

Fig. 1 shows a schematic GSTWH. The tank losses are complex, with the dominant thermal loss pathways shown on the left. Thermal shorts and bottom skin losses are challenging to model. Errors in the test (2) cause up to 20% error in UA_{tank} for GSTWHs, and up to 40% error for ESTWHs (4). Most units have a pilot, which is generally ignored in dynamic modeling and is subsumed here as part of the gas usage. This practice should be changed for simulated ratings, as the pilot light may cause the tank to go over setpoint and waste significant energy, especially when used with a SWH supplying very hot water at times (9).

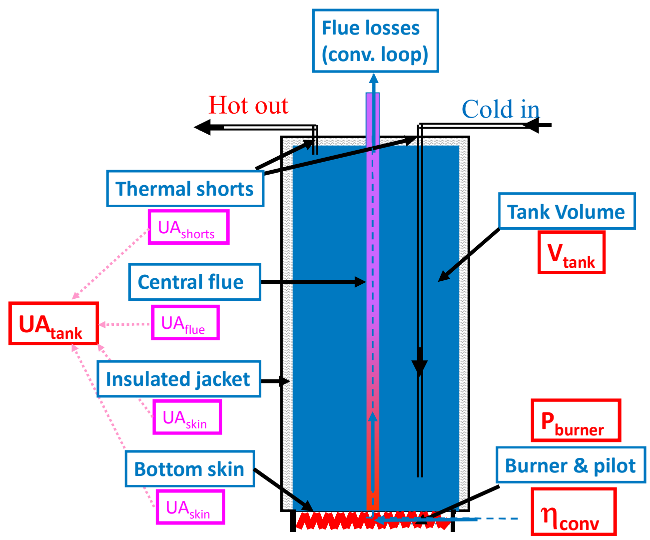

Fig. 1. Schematic gas storage tank water heater.

3.1 Model derivation

A one-node thermal circuit model for a GSTWH is shown in Fig. 2. Energy factor data shows storage tanks are relatively uniform in temperature (10). The tank instantaneous one-node (isothermal) energy balance is written as:

$$C dT_{tank}/dt = \eta_{conv}\dot{q}_{gas,in} - UA_{tank}\Delta T_{tank\text{-}env} - \dot{m}_{draw}c_p\Delta T_{out\text{-}in} \quad (3)$$

Integrating over a year, neglecting any change in tank temperature at time endpoints, and re-arranging, we have:

$$Q_{GSTWH,year}=(M_{draw,year}c_p\Delta T_{set\text{-}in}/\eta_{conv})*$$
$$(1 + UA_{tank}\Delta T_{tank\text{-}env}\Delta t_{year}/M_{draw,year}c_p\Delta T_{set\text{-}in}) \quad (4)$$

Key GSTWH parameters can be inferred as in (4):

$$UA_{tank}=(RE/EF-1)/[\Delta T_{tank\text{-}env}(\Delta t_{day}/Q_{out,day}-1/(P_{in}EF))] \quad (5)$$

$$\eta_{conv} = RE + UA_{tank}(\Delta T_{tank\text{-}env})/P_{max}. \quad (6)$$

Similar formulae exist for ESTWHs (11), with the key differences that $\eta_{conv,elec} = 1.0$, the listed RE_{ESTWHs} is meaningless, the actual RE must be calculated from Eqn. 6, and the tank is generally stratified with a cold slug below the lower electric heating element.

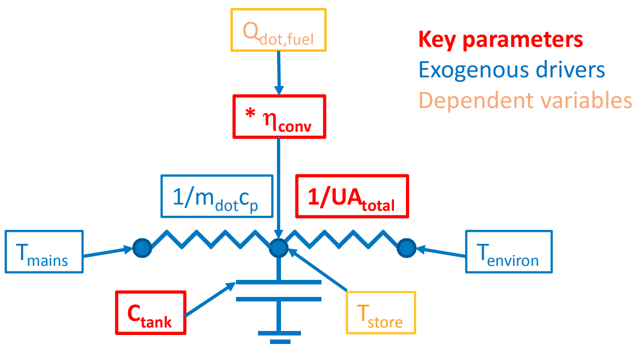

Fig. 2. Thermal circuit one-node model for a gas storage tank water heater. The model also applies to a tankless water heater, understanding C_{tank} is $C_{TWH\text{-}hx}$ and T_{store} is $T_{TWH\text{-}hx}$.

3.2 <u>Model validation with field monitoring data</u>

To validate the model and parameter extraction process, we obtained data from a recent project monitoring WHs (12), including data for $T_{env,avg}$ and T_{set}. Fig. 3 shows the daily energy factor (EF_{day}) from one site, with a comparison to model predictions using the tank's rating data to calculate the key parameters using Eqs. 5-6. The rated EF_{test} of the installed GSTWH was 0.60. The predictions were systematically high, and EF_{test} was lowered to 0.58 to improve the fit. EF_{day} for the GSTWH falls off with lowered load similarly in models and data.

4. <u>TANKLESS WATER HEATERS (TWHs)</u>

A schematic of a TWH is shown in Fig. 4. The heat exchanger thermal mass consists of the contained water, and metal piping and fins. TWH features which are neglected include: i) the *minimum flow* of 0.5-0.8 gpm to activate the burner, to prevent overshooting the setpoint; ii) the *safety delay* of several seconds as air flow rates are established and confirmed before ignition; and iii) *freeze protection* energy. Standby power consumption is included in final computations. We note that the analysis here assumes that η_{conv} is constant, and so results here do not strictly apply to condensing tankless where η_{conv} varies. However, a reasonable estimate can be gotten TWH commonly output water below setpoint at the draw start. ***Water delivered below setpoint is assumed not used, so $Q_{gas,in}$ and $Q_{TWH,out}$ when $T_{TWH,out} < T_{set}$ are considered wasted.***

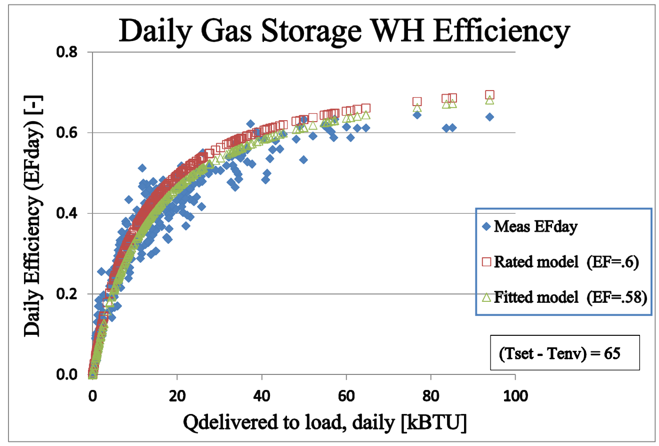

Fig. 3. Daily energy factor data, compared to the simple GSTWH model w/ inferred parameters using two EFtest values, published ratings (0.6) and a visual fit (0.58).

4.1 <u>TWH ratings data</u>

The TWH ratings data include: 1) the maximum draw flow meeting set point, $\dot{m}_{draw,max}$; 2) the maximum gas input rate, $\dot{m}_{gas,max}$; 3) the energy factor EF; 4) the recovery efficiency RE; and 5) the storage volume V_{tank} (1). Only RE and $\dot{m}_{gas,max}$ are useful values. V_{tank} is set to zero for all units, making it meaningless.[1] Ratings data[2] indicate EF is equal to RE within noise. RE is a good measure of η_{conv}, as the 3-min draw is long enough to swamp the effect of the wasted energy as temperature ramps up. The value of the maximum water flow $\dot{m}_{draw,max}$ and the maximum gas input $\dot{q}_{max,gas}$ are essentially redundant.

4.2 <u>TWH Model</u>

The model for a TWH is based upon analysis of a draw event, solving for $T_{TWH}(t)$. A draw event is illustrated in Fig. 5, surrounded by end of the previous draw and start of the next draw. The draw is analyzed into three stages:

[1] Setting V_{tank} =0 is a misleading gaffe because it is exactly *non-zero* volume that causes TWH inefficiencies. "NA" would be preferable.
[2] Taking one manufacture's data for 191 TWH models, (RE_i-EF_i) had mean bias of -0.002, and RMS deviation of 0.007. Measurement error in $(RE, EF)_{TWH}$ is ~.01, so differences are well within error limits.

- Ramp-up. The draw starts and gas comes on to bring the flowing water to set-point.
- Steady state. The outlet temperature is stable at T_{set}, with $q_{gas,in}$ adjusted to meet the setpoint.
- Temperature decay. The sensible energy left in the heat exchanger at draw end decays to ambient.

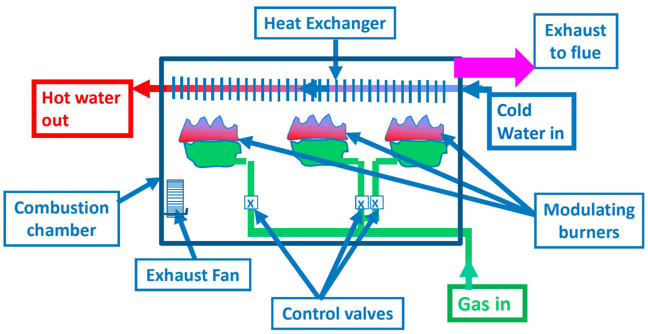

Fig. 4. A schematic of a tankless water heater.

The TWH thermal circuit model is identical to Fig. 2 (with relabeling), so the energy balance for a TWH is:

$$CdT_{TWH}/dt = \eta_{conv}\dot{q}_{gas,in} - UA_{TWH}\Delta T_{tank\text{-}env} - \dot{m}_{draw}c_p\Delta T_{out\text{-}in} \quad (7)$$

Gas flows during two time periods, Δt_{ramp} (with $T_{out} < T_{set}$) and Δt_{steady} (with $T_{out} = T_{set}$). During ramp up, we assume $\dot{q}_{gas} = f_{ramp}\dot{q}_{gas,max}$, where $f_{ramp} < 1$. f_{ramp} represents the modulation by the TWH controls to avoid significantly overshooting the setpoint; for this paper, we take f_{ramp} = 0.255, as determined from calculations with the model in (11). We want to solve for the draw efficiency η_{draw}:

$$\begin{aligned}\eta_{draw} &\equiv Q_{to\text{-}load}/Q_{gas,total} = Q_{to\text{-}load}/(Q_{to\text{-}load}/\eta_{conv} + Q_{gas,ramp}) \\ &= \eta_{conv}/[1 + \eta_{conv}\,(Q_{gas,ramp}/Q_{to\text{-}load})] \qquad (8)\end{aligned}$$

Decay from previous draw. The time since the last draw, Δt_{prev}, is considered given. It is used to compute T_{TWH} at the start of the draw being analyzed. Solving eq. 7 with $\dot{q}_{gas,in} = \dot{m}_{draw} = 0$, we have:

$$T_{start} = T_{env} + (T_{set} - T_{env})exp(-\Delta t_{prev}/\tau_{decay}), \text{ where} \quad (9)$$
$$\tau_{decay} = C_{TWH}/UA_{TWH}$$

Ramp up to setpoint. We solve Eq. 7 with all terms non-zero, and then solve for the time to reach T_{set}:

$$T_{set} = T_{t=\infty} + (T_{start} - T_{t=\infty})exp(-\Delta t_{ramp}/\tau_{ramp}), \text{ where:} \quad (10)$$
$$T_{t=\infty} = K1/K2, \text{ where:}$$
$$K1 = \eta_{conv}\dot{q}_{gas,max} + UA_{TWH}T_{env} + \dot{m}_{draw}c_pT_{in,mains},$$
$$K2 = UA_{TWH} + \dot{m}_{draw}c_p,$$
$$\tau_{ramp} = C_{TWH}/(UA_{TWH} + \dot{m}_{draw}c_p)$$
$$\Rightarrow \Delta t_{ramp} = \tau_{ramp}ln[(T_{t=\infty} - T_{start})/(T_{t=\infty} - T_{set})] \quad (11)$$

Steady state, at setpoint: The time duration of the steady state period $\Delta t_{steady\text{-}state}$ is the period the homeowner uses the hot water. It is an exogenous variable considered known. The total gas input during the steady state period is:

$$Q_{gas,steady} = Q_{to\text{-}load}/\eta_{conv} = V_{load}\rho_{water}c_p\Delta T_{out\text{-}in}/\eta_{conv} \quad (12)$$

Eqs. 8-12 provide an exact solution for the draw efficiency. Results using the solver (13) are shown in Fig. 6.

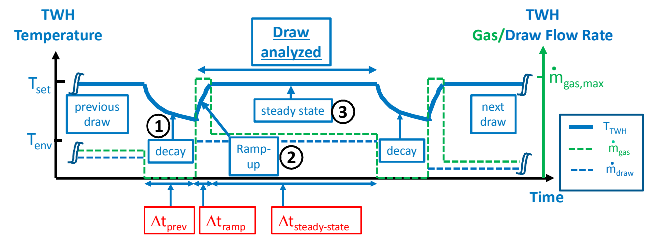

Fig. 5. Schematic of a draw, defining time periods characteristic of each draw event.

The ratings data provide a value for η_{conv}. The values for the other two key characteristics C_{TWH} and UA_{TWH} have to be obtained elsewhere. The unit can be tested specifically to provide these parameters, as was done in (14). If test data is available for a unit that is similar to the unit of interest, the values determined in the test can be used. The gas water heater characteristics in (14) were used for all calculations here: $C = 9$ kJ/C, $UA = 3.6$ W/C. Alternatively, the values can be estimated from their definition and a physical description of the unit (pipe/fin materials, and geometry of the combustion chamber and outside enclosures).

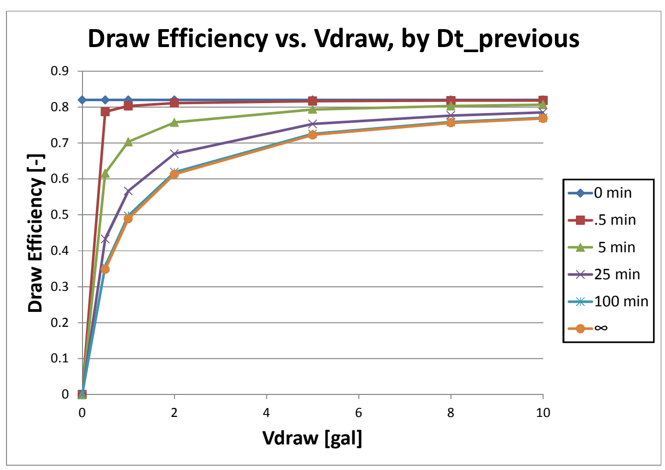

Fig. 6. Draw efficiency as a function of draw volume, for various values of the delay since the last draw.

Annual energy is to be computed for some chosen representative draw pattern, such as one of the annual-period draw patterns published in (15). Annual energy consumption is written as the sum over all draws:

$$Q_{gas,year} = \Sigma_d V_{draw,d} \rho c_p \Delta T_{set\text{-}in} / \eta_{draw,d} \quad (13)$$

To collapse the sum, we divide the draws into bins in the 2-D space (Δt_{prev}, V_{draw}). Table 1 shows a suggested partition that is used below to do comparisons with field data. The volume of draw in each of the bins ($\Delta t_{prev,i}$, $V_{draw,j}$) is to be computed from a chosen draw pattern. This is done only once, and used for all TWHs using that draw specification. The efficiency of the draws in each bin is given by Eqs. 8-12 at the chosen representative values of ($\Delta t_{prev,i}$, $V_{draw,j}$) in each i,j bin. The annual energy consumption is then computed by summing over the bins:

$$Q_{gas,year} = \Sigma_{i,j} V_{draw,i,j} \rho c_p \Delta T_{set\text{-}in} / \eta_{draw,i,j}, \quad \text{where} \quad (14)$$

$V_{draw,i,j}$ is the draw volume in bin i,j
$\eta_{draw,i,j}$ is the draw efficiency in bin i,j

The calculated draw efficiencies as a function of draw volume for the Table 1 delay times are shown in Fig. 6. The figure illustrates clearly how TWH degrade in efficiency from the rated EF_{test}, especially with longer delays and for smaller draws.

TABLE 1. NORMALIZED DRAW BINS, $v_{draw,ij}$

	Δt_{prev} (min)[1] ↓			
Vdraw (gal)[1] ↓	< 1 (0.5)	1-10 (5)	10-40 (25)	>40 (100)
0-1 (0.5)	0.18[2]	0.09[2]	0.015[2]	0.015[2]
1-3 (2)	0.3[2]	0.15[2]	0.025[2]	0.025[2]
> 3 (5)	0.12[2]	0.06[2]	0.01[2]	0.01[2]

1. The value to use for calculations is shown in parentheses
2. Values are for the ersatz draw analyzed here, with medium use

4.3 Model validation.

The TWH model was validated using EF_{day} data from (12). Fig. 7 shows EF_{day} from one site when that site used a non-condensing TWH with EF=0.82. No data were available other than EF_{day}, T_{set}, and $T_{env,avg}$. We thus made up an algorithm to fill the matrix, an ersatz draw but a reasonable surrogate for detailed data, accepting as a result that we can expect to match the general trend only.

TABLE 2. EFFICIENCIES IN DRAW BINS $\eta_{draw,i,j}$

	Δt_{prev}			
V_{draw} ↓	0.5 min	5 min	25 min	100 min
0.5 gal	0.79	0.62	0.43	0.36
2 gal	0.81	0.76	0.67	0.62
5 gal	0.82	0.79	0.75	0.73

We start by deriving a normalized volume matrix $v_{draw,ij}$, which gives the fraction in each bin of one unit of draw volume, where i labels Δt_{prev} bins and j labels V_{draw} bins. Defining the single-variable distributions $v_{draw,j} \equiv \Sigma_i v_{draw,ij}$ and $v_{draw,j} \equiv \Sigma_i v_{draw,ij}$, for a given choice of these bin values we derive $v_{draw,ij}$ using the rule

$$v_{draw,ij} = v_{draw,i} v_{draw,j} \quad (15)$$

Using $\Sigma_{ij} v_{draw,ij} = 1$, we note $\Sigma_i v_{draw,i}, = \Sigma_j v_{draw,j} = 1$, as in Table 3 values, which is a useful constraint on choosing values. The actual draw volumes and energy in each bin are gotten by:

$$V_{draw,ij} = V_{draw,day} v_{draw,ij}, \text{ and}$$
$$Q_{draw,ij} = \rho c_p V_{draw,day} \Delta T v_{draw,ij} \quad (16)$$

Guidance for creating the Δt_{prev} and ersatz distributions is gotten by noting: 1) the Δt_{prev} distribution shrinks toward smaller values with larger daily draw volume (more draws/day,...), and 2) V_{draw} will tend to be smaller with smaller daily draw (fewer showers and baths,...). We thus divide loads into small, medium, and high volume levels. Table 3 shows the three levels of $V_{draw,day}$ that we used , along with a chosen representative daily draw volume. Table 4 shows the values for $v_{draw,i=\Delta tprev}$ and $v_{draw,j=Vdraw}$ used for calculations here. The values $v_{draw,ij}$ shown in Table 1 for the medium use case were derived using Eq. 15 with Table 4 medium use values.

TABLE 3. LOW, MEDIUM, AND HIGH USE CASES

Draw Volume Levels (gal/day)		
Low Use	Med Use	High Use
0-15 (8)	15-50 (30)	>50 (60)

TABLE 4. BIN PROBABILITIES

Δt_{prev} binned distribution				
	.5 min	5 min	25 min	100 min
Low Use	0.3	0.4	0.2	0.1
Med Use	0.6	0.3	0.05	0.05
High Use	0.7	0.2	0.08	0.02

V_{draw} binned distribution			
	0.5 gal	2 gal	5 gal
Low Use	0.7	0.1	0.2
Med Use	0.3	0.5	0.2
High Use	0.3	0.3	0.4

The resulting model predictions are plotted in Fig. 7 at the 3 volume/load points of Table 3 (in addition to the zero draw point). The model systematically under-predicts the measured data; however, the trend of slow decrease with decreasing load (until the sharp decrease at roughly 10 kBtu/day) is reproduced. The under-prediction may be due to the ersatz bin choices. More work is needed to resolve the source of the discrepancy and better demonstrate model validity. Since the simple model solves analytically the same energy equations as numerical simulation models as in (14), the simple model can serve as a basis for a new, faster time-step simulation that is based upon using Eqs. 8-12 for each draw, stepping through draws rather than second-level time steps.

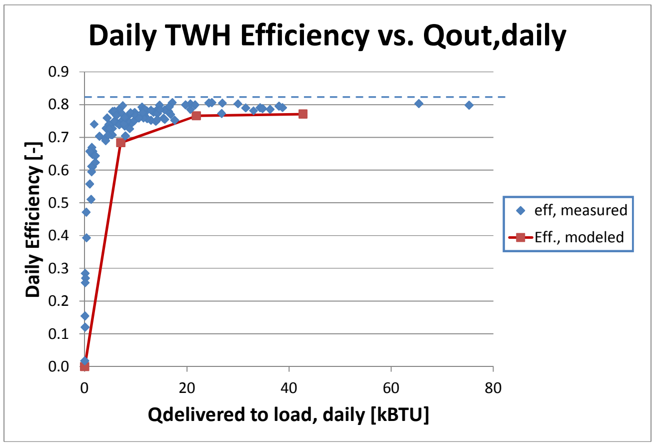

Fig. 7. EF_{day} data (9), observed (points) and simulated (line).

5. CONCLUSIONS AND FUTURE WORK

For two water heater types covering most installed units, we have shown that simple algebraic models combined with ratings data facilitate simple and reasonably-accurate, model-specific prediction of the annual performance of most water heater models. This simplifies analyses such as: economic analyses for consumers, policy-makers, and incentive organizations; tradeoffs amongst units having different cost and performance; and sizing optimization for given loads.

Storage tank water heaters are a paradigm of simple models using ratings: the simple model is reasonably rigorous and all the key parameters are derived from the ratings data. The model matches the drop-off in efficiency of gas storage tank water heaters observed in field data. Ideally the process would be validated over a wide range of units with laboratory data. This would be the necessary if this method is used to convert previous ratings data under (2) to the expected results under the new test conditions (4).

A model has been developed for tankless water heaters (TWHs), based upon analytical solutions for the three stages of a draw. The draw efficiency depends on the time since the previous draw and the draw volume. Annual consumption is the sum over all draws, with bins of ($\Delta t_{prev,i}$, $V_{draw,j}$) used to collapse the sums. The models for GSTWHs and TWHs account for the differing drop-off of daily efficiency with decreasing daily draw volume, seen in field data. More work is needed to fully validate the models.

A similarly-simple model has not been developed yet for a heat pump water heater (HPWH). The unsolved problem is how to correlate the electric element use (16), which lowers efficiency significantly ($\eta_{conv,elec} = 1$, $\eta_{conv,HPWH} = \sim 2$). A correlation might best use discrete laboratory testing where the data can be unambiguous. It is unclear how many distinctly different control strategies there are to be modeled. A proposed modified EF test could serve as a simple HPWH model if the specified data were available (17).

A simple model for solar water heaters (SWHs) is also needed. There have been correlation models for SWH performance for many years (5). These models are suitable for simple methods like here, but need work in order to know how to relate these models to available annual ratings data. Although an iteration could be applied to fit annual consumption at a site, it is unclear how accurate that method would be. The *constant efficiency* method (18, 19) for solar water heaters could be used with ratings data directly, but further work is needed to clarify the method's applicability over various climates and available systems.

The long-term goal of this work is to produce a simple tool providing water heater annual energy consumption, based on fast, simple algebraic algorithms such as developed here. Simple heat pump and solar water heating models should be included. The tool will provide guidance on how to estimate model inputs using ratings data and other means. The tool could be the basis for mobile apps for quick in-store decision support or for whole-house tools used for home ratings.

6. NOMENCLATURE

Symbols and Acronyms

A	Area, of solar collector or storage tank
C	Thermal capacitance
c_p	Specific heat
EF	Energy factor from DOE standard test
ESTWH	Electric storage tank water heater
GSTWH	Gas storage tank water heater
i,j	Labels for ($\Delta t_{prev,i}$, $V_{draw,j}$)
$\dot{m}$	Mass flow rate (mass/time)
M	Mass of water drawn over some period
P	Thermal power into tank
$\dot{q}$	Thermal energy flux [Energy/time]
Q	Quantity of thermal energy [Energy]
SWH	Solar Water Heater

RE	Recovery efficiency from DOE standard test
T	Temperature
t	Time
TWH	Tankless water heater
UA	Total loss coefficient
v	Specific volume (volume per gallon)
V	Volume

Greek symbols

Δ	Difference of
η	Efficiency

Subscripts

avg	Average
conv	Conversion of input energy to thermal energy
day	Time interval of one day
decay	Decay of temperature at end of a draw
draw	Draw of hot water
elec	Electric water heater
env	Environment surrounding tank
GSTWH	Gas storage tank water heater
gas	Natural gas (or any fossil fuel)
hx	Heat exchanger in a tankless water heater
i	Index for Δt_{prev} bin
in	Input to the tank (fuel or inlet water)
j	Index for V_{draw} bin
k	Kind of backup water heater for solar preheat
load	Delivered energy to the load
loss	Losses from tank to environment
mains	Mains water inlet temperature
ramp	The ramp from T_{start} to T_{set} at start of a TWH draw
out	Out of the water heater
out-in	Temperature difference, T_{out} - T_{in}
previous	Previous draw
set-env	Temperature difference, $T_{set} - T_{env}$
set	Effective set point
start	Starting value of a variable
SWH	Solar water heater
tank	Storage tank water heater
TWH	Tankless water heater
WH	Water heater
year	Year-long time period

7. ACKNOWLEDGMENTS

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8. REFERENCES

(1) Air Conditioning, Heating and Refrigeration Institute (AHRI) publishes ratings on water heaters and other appliances at: http://www.ahridirectory.org/ahridirectory.

(2) The DOE Standard Energy Factor test is described at: Federal Register Vol. 63, part 90 p. 25995, 10CFR 430.

(3) TRNSYS is available from Thermal Energy System Specialists, 22 North Carroll Street - Suite 370, Madison, WI, www.tess-inc.org.

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(5) FCHART is available from FCHART Software, Box 44042, Madison, WI 53744

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