SOLAR 2010:

THREE DIMENSIONAL MATHEMATICAL MODEL OF TIME –DEPENDENT CONVECTION OF FLUIDS IN ENERGY SYSTEMS THAT USE SOLAR RADIATION

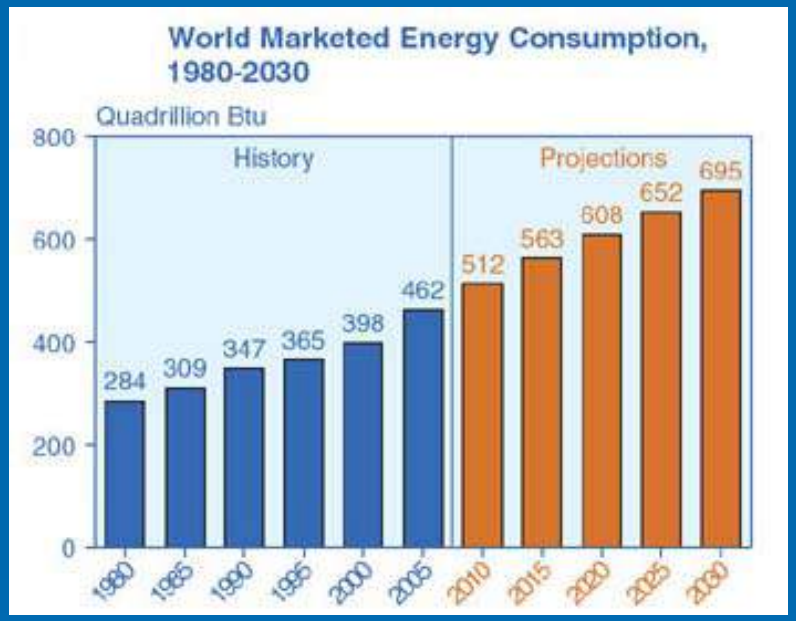
Rafael O. Castro, John Duffy

U Mass Lowell

OUTLINE

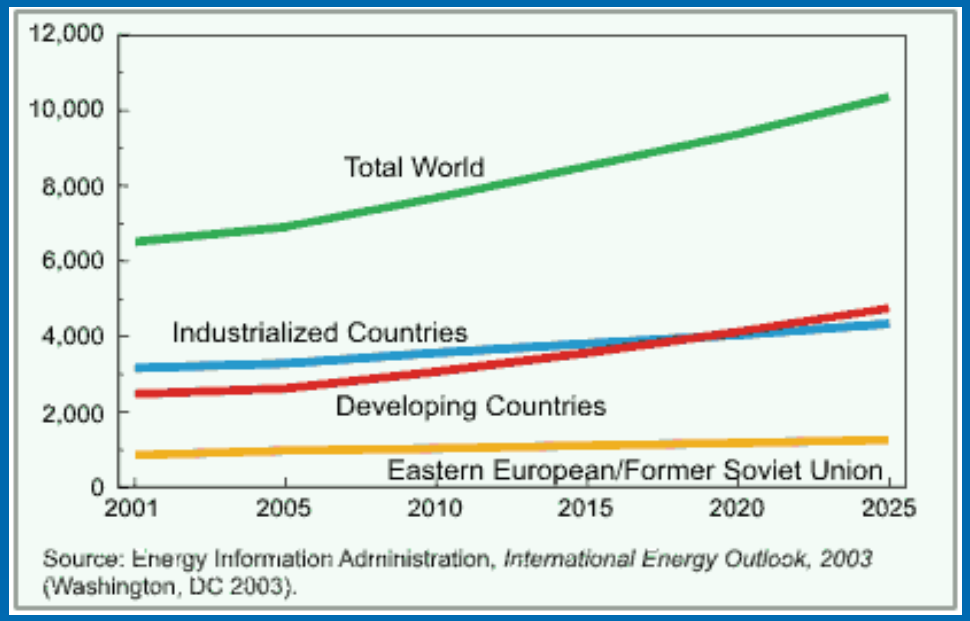
1. Introduction
2. Modeling of Convection of fluids
3. Numerical Method
4. Application Example
5. Design Optimization
6. Concluding Remarks

World Energy Demand



Source: Official Energy Statistics from the U.S. Government, 2008

World CO_2 Emissions

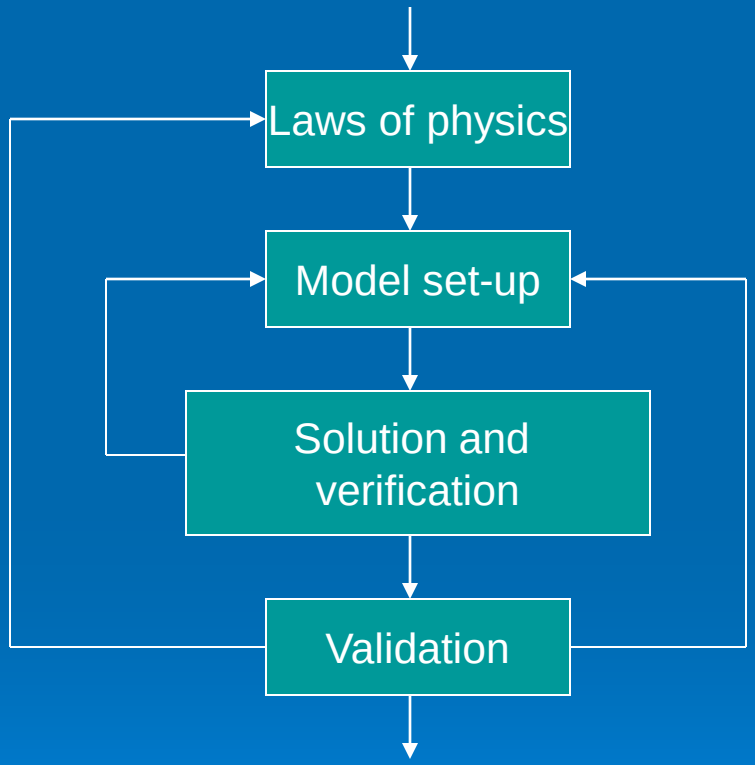


Objective:

To present an overview of the main aspects for the mathematical modeling of convection of fluids that are present in a wide number of energy systems.

The mathematical modeling process:

Credit: *Data* - Hubble Legacy Archive, ESA, NASA *Processing* - Robert Gendler

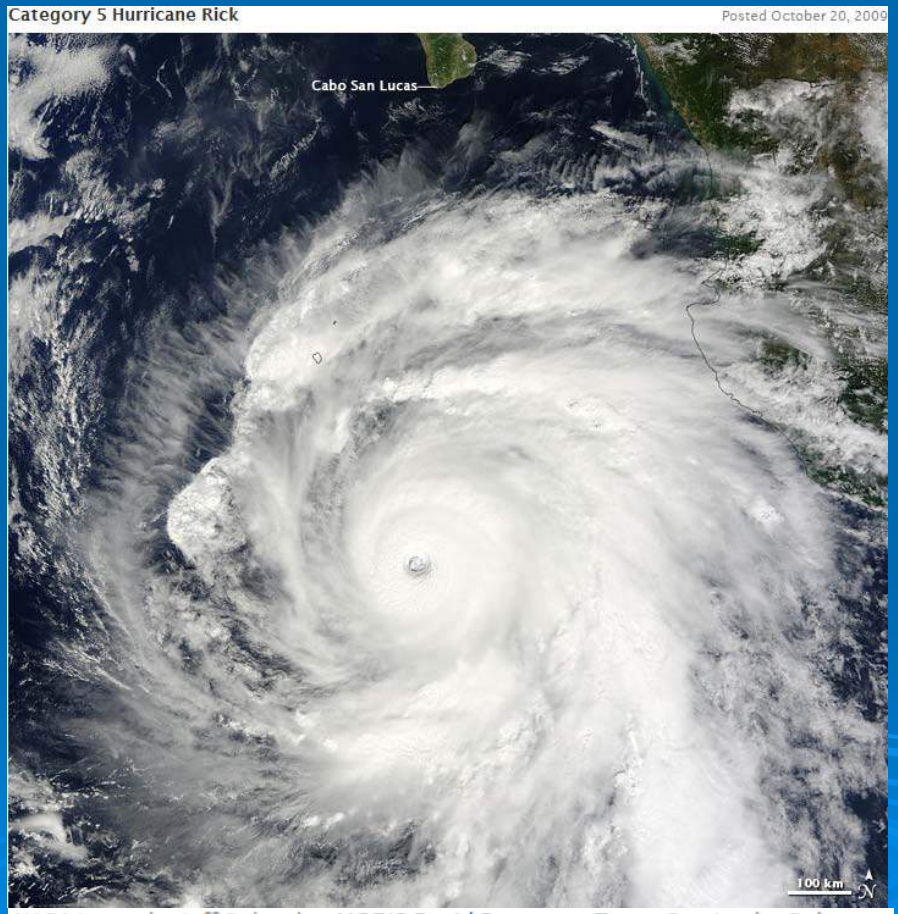


source: nasa

What are the governing equations of fluid flows?

The Deployment of a Large Liquid Column in Low-gravity

source: http://spaceflightsystems.grc.nasa.gov



NASA image by Jeff Schmaltz, MODIS Rapid Response Team. Caption by Rebecca Lindsey, NASA Earth Observatory.

Governing equations:

Conservation of mass

$$\frac{\partial \rho}{\partial t} + \nabla.(\rho \vec{V}) - 0 - 0 = 0$$

Conservation of momentum

$$\rho \frac{\partial \vec{V}}{\partial t} + \rho \vec{V}.\nabla \vec{V} + \nabla p + \nabla.\overleftrightarrow{\tau} + \vec{f} = 0$$

Conservation of energy

$$\rho C_p \frac{\partial T}{\partial t} + \rho C_p \vec{V}.\nabla T + \nabla.(-K\nabla T + \overrightarrow{q_r}) + \left(\frac{\partial ln\rho}{\partial lnT}\right)_p \frac{Dp}{Dt} + Q_J - Q_r = 0$$

Diffusive fluxes

$$\overleftrightarrow{\tau} = [-\mu(\nabla \vec{V} + (\nabla \vec{V})^{\dagger} + (\frac{2}{3}\mu - \kappa)(\nabla.\vec{V})\overleftrightarrow{\delta}$$

$$\overrightarrow{q'} = -K\nabla T + \overrightarrow{q'_r}$$

The K-epsilon Model

Turbulent flows are always dissipative. Viscous shear stresses perform deformation work which increases the internal energy of the fluid at the expense of kinetic energy of turbulence.

K-epsilon:

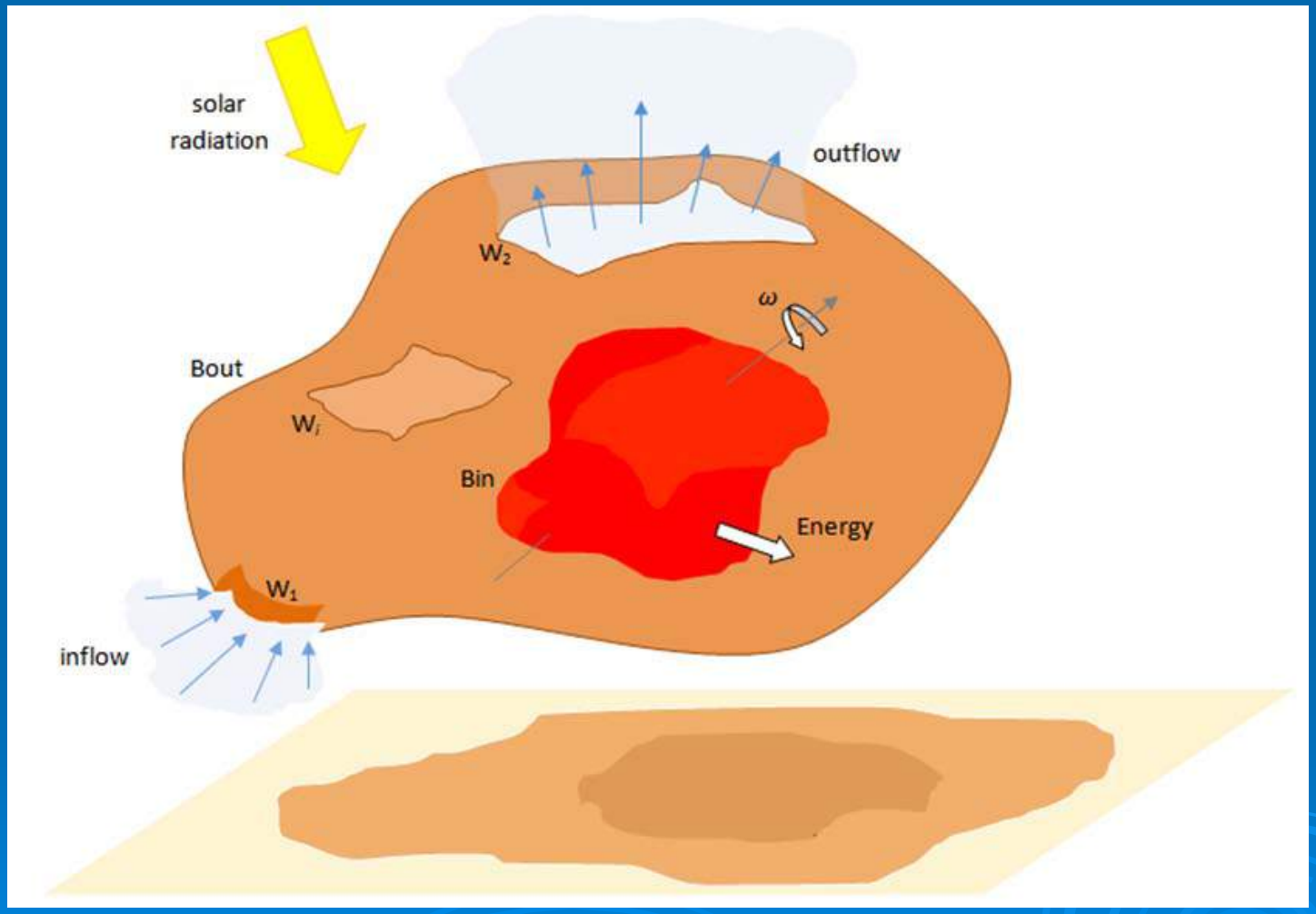
Equation for the turbulent kinetic energy κ:

$$\frac{\partial}{\partial t}(\rho\kappa) + \frac{\partial}{\partial x_i}(\rho\kappa u_i) = \frac{\partial}{\partial x_j}\left[\left(\mu + \frac{\mu_t}{\sigma_\kappa}\right)\frac{\partial\kappa}{\partial x_j}\right] - \rho\in + \tau_{ij}\frac{\partial u_i}{\partial x_j},$$

Equation for the turbulent kinetic energy dissipation rate , ε:

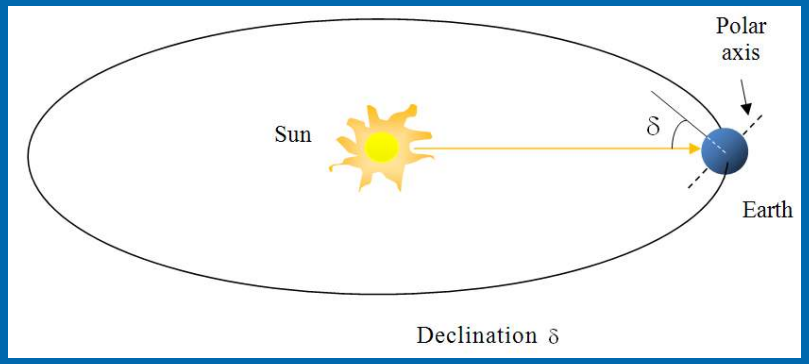
$$\frac{\partial(\rho\in)}{\partial t} + \frac{\partial(\rho\in u_i)}{\partial x_i} = \frac{\partial}{\partial x_j}\left[\left(\mu + \frac{\mu_t}{\sigma_\in}\right)\frac{\partial\in}{\partial x_j}\right] + C_{1\in}\frac{\in}{\kappa}\tau_{ij}\frac{\partial u_i}{\partial x_j} - C_{2\in}\rho\frac{\in^2}{\kappa}.$$

$$\mu_t = \rho C_\mu \frac{\kappa^2}{\in}$$

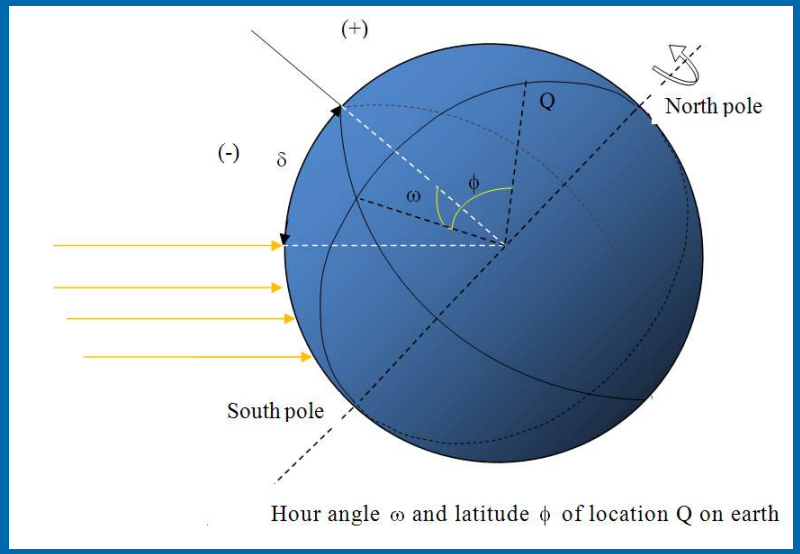


Three dimensional representation of energy systems that use solar radiation

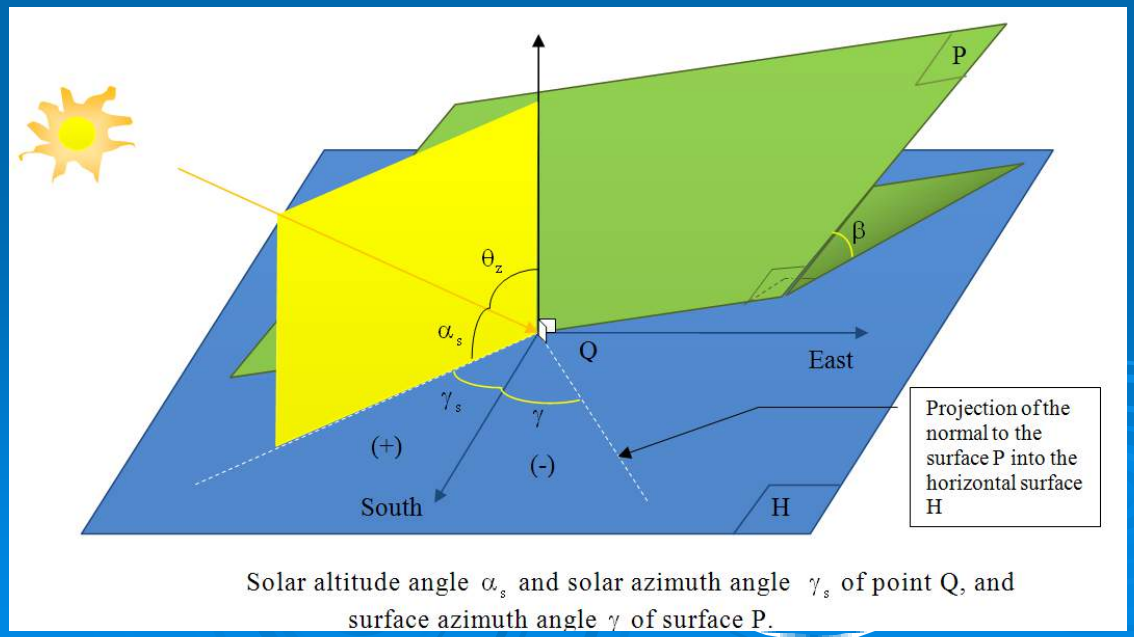
Solar radiation:



Declination δ

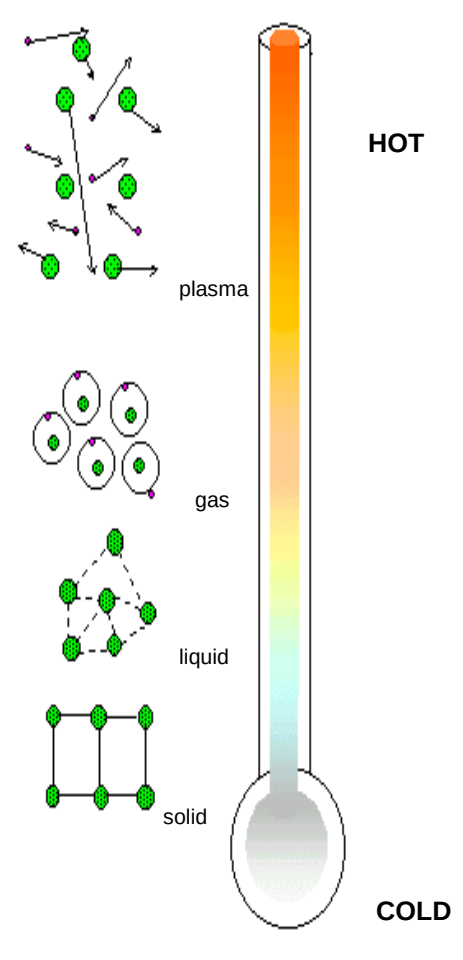


Hour angle ω and latitude ϕ of location Q on earth



Solar altitude angle α_s and solar azimuth angle γ_s of point Q, and surface azimuth angle γ of surface P.

Thermodynamic and Transport Properties



States of matter

Specific heat:

- Specific heat at constant pressure

$$c_P = \frac{T}{N}\left(\frac{\partial S}{\partial T}\right)_P = -\frac{T}{N}\frac{\partial^2 G}{\partial T^2}$$

- Specific heat at constant volume

$$c_V = \frac{T}{N}\left(\frac{\partial S}{\partial T}\right)_V = -\frac{T}{N}\frac{\partial^2 A}{\partial T^2}$$

Coefficient of thermal expansion:

$$\alpha = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_P = \frac{1}{V}\frac{\partial^2 G}{\partial P \partial T}$$

Compressibility:

- Isothermal compressibility

$$\beta_T = -\frac{1}{V}\left(\frac{\partial V}{\partial P}\right)_T = -\frac{1}{V}\frac{\partial^2 G}{\partial P^2}$$

- Adiabatic compressibility

$$\beta_S = -\frac{1}{V}\left(\frac{\partial V}{\partial P}\right)_S = -\frac{1}{V}\frac{\partial^2 H}{\partial P^2}$$

$$R_g = K_g \frac{\sum_s n_s}{\sum_s n_s m_s} \qquad \partial\rho = \rho_{,p}\,\partial p + \rho_{,T}\,\partial T \qquad \rho = \frac{p}{R_g T}$$

Solver Layout

Loop: Time stepping

$$\mathbf{Res}(t, \mathbf{X}, \mathbf{Y}, \dot{\mathbf{Y}}) = 0 \quad \rightarrow \quad \mathbf{Res}(\mathbf{Y}) = 0$$

-Second order implicit predictor- multicorrector
(Jansen,2000)

Loop: Solution Non-linear System

$$\left\| \mathbf{Res} + \mathbf{Jac}\Delta\mathbf{Y}^{k+1} \right\| \leq \eta \left\| \mathbf{Res} \right\|$$

$$\mathbf{Y}^{k+1} = \mathbf{Y}^{k} + \lambda\Delta\mathbf{Y}^{k+1}$$

-Globalized Newton-Krylov method
(Kelly,2003)

Loop: Solution Linear System

-Preconditioned Generalized Minimal residual (GMRS)
(Scaling, Pre-preconditioning, Preconditioning)

$$\mathbf{A}\mathbf{x} = \mathbf{b}$$

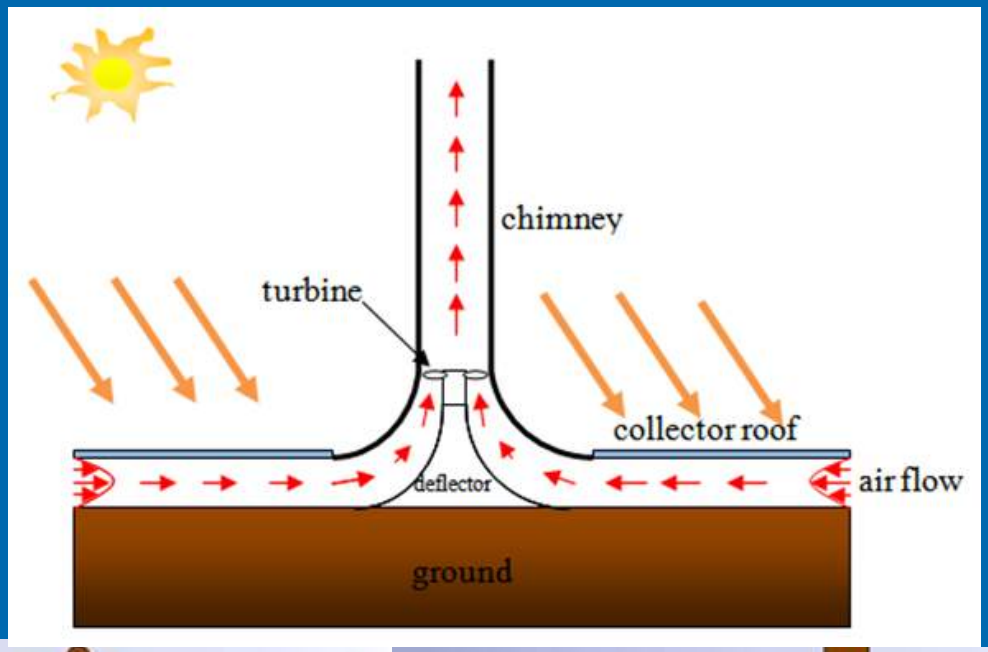
$$\left(\mathbf{P}^{-1}\mathbf{A}\right)\mathbf{x} = \left(\mathbf{P}^{-1}\mathbf{b}\right)$$

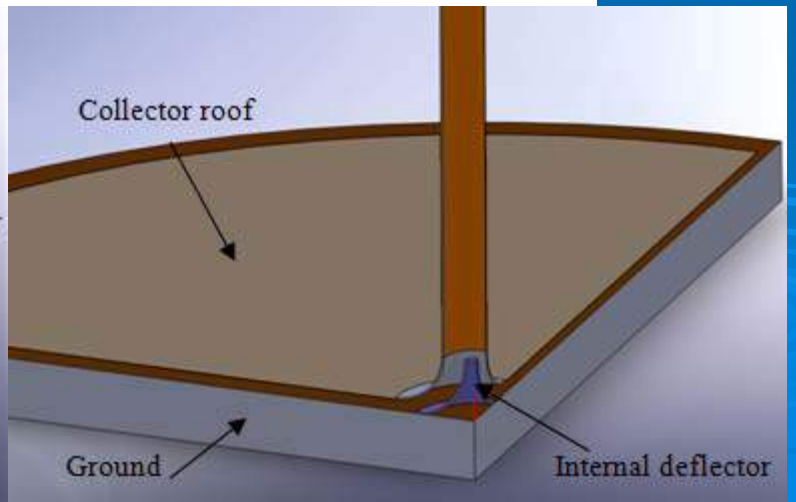
end

end

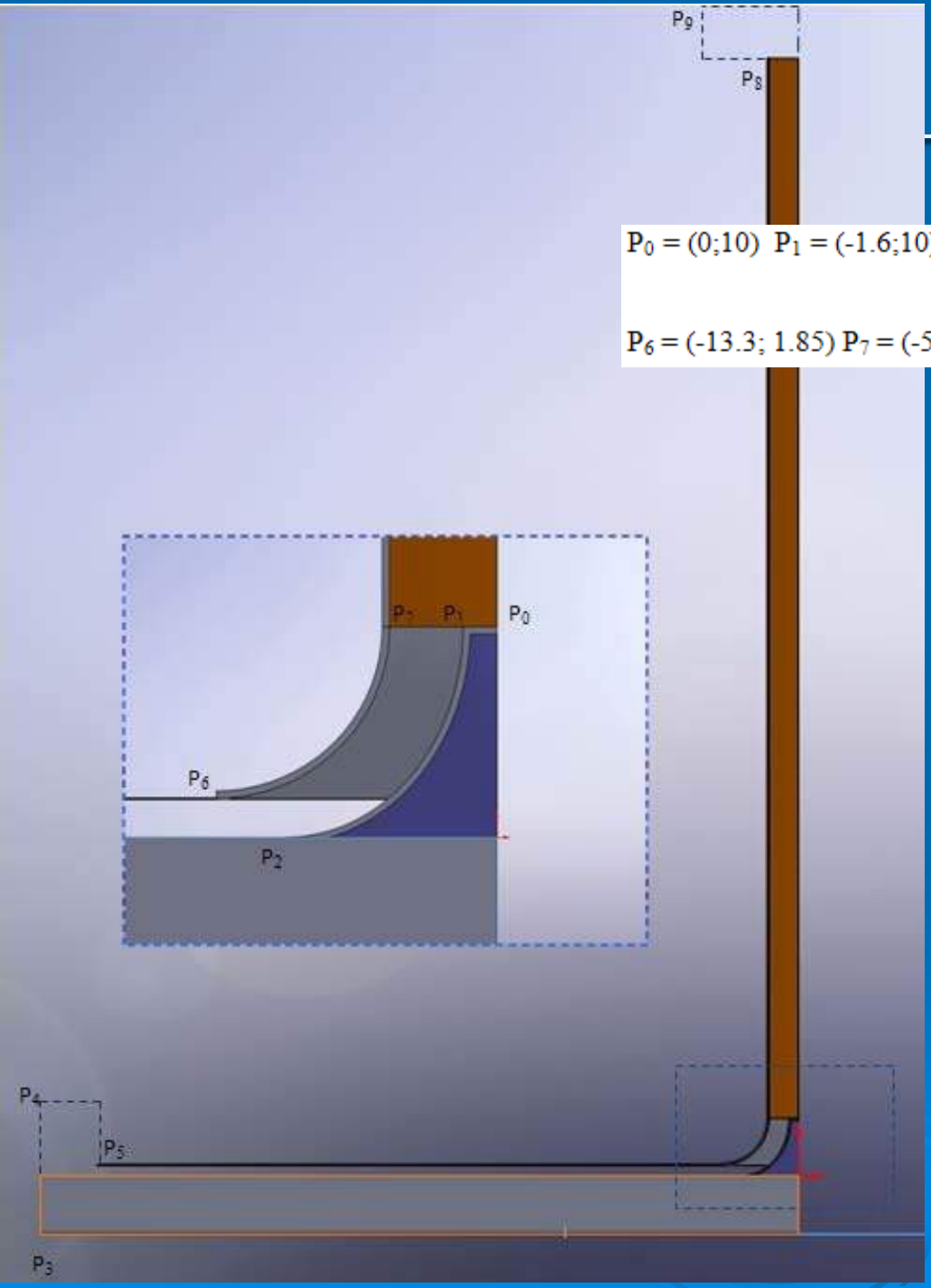
end

Solar Updraft Tower





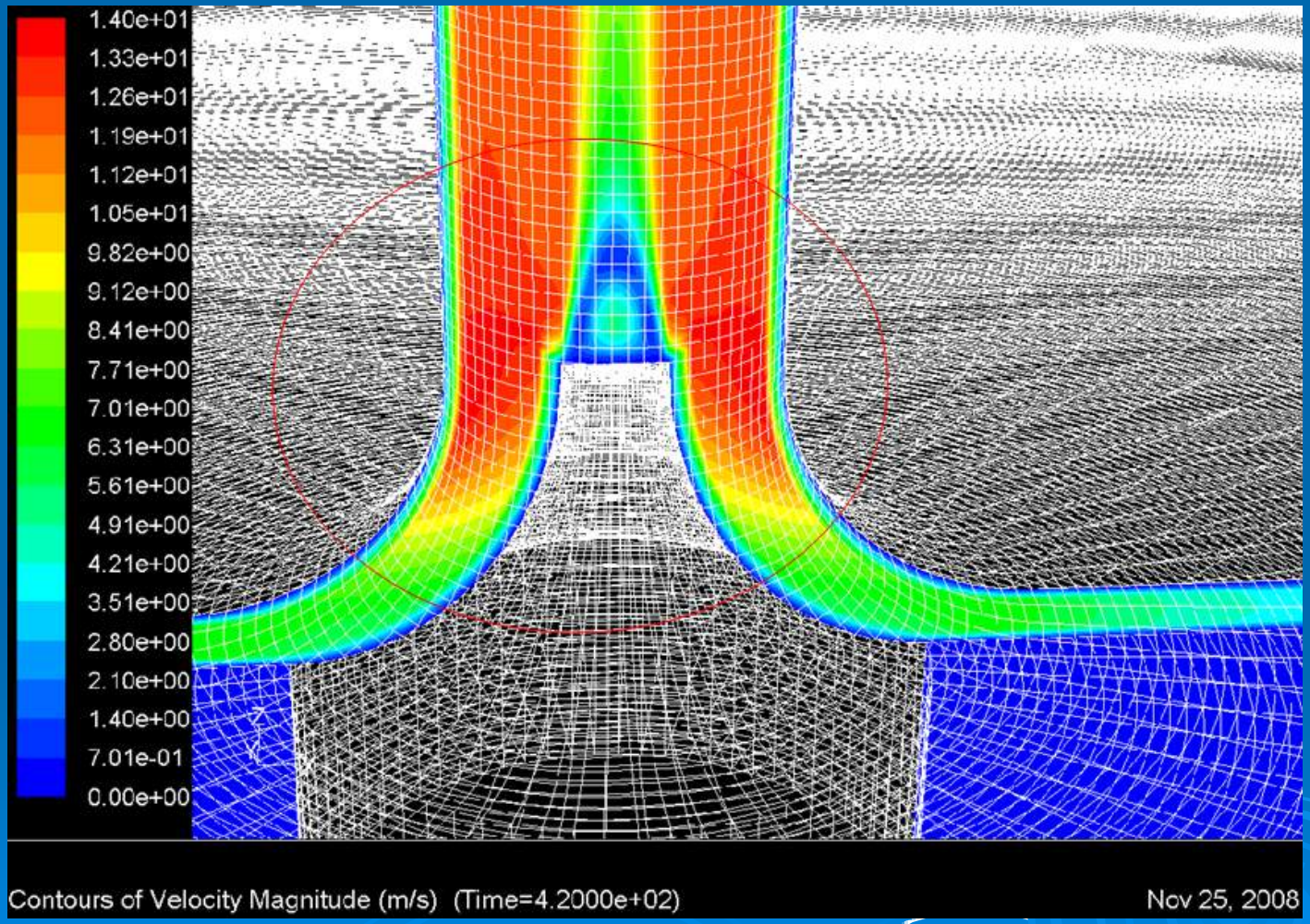
4. Application example

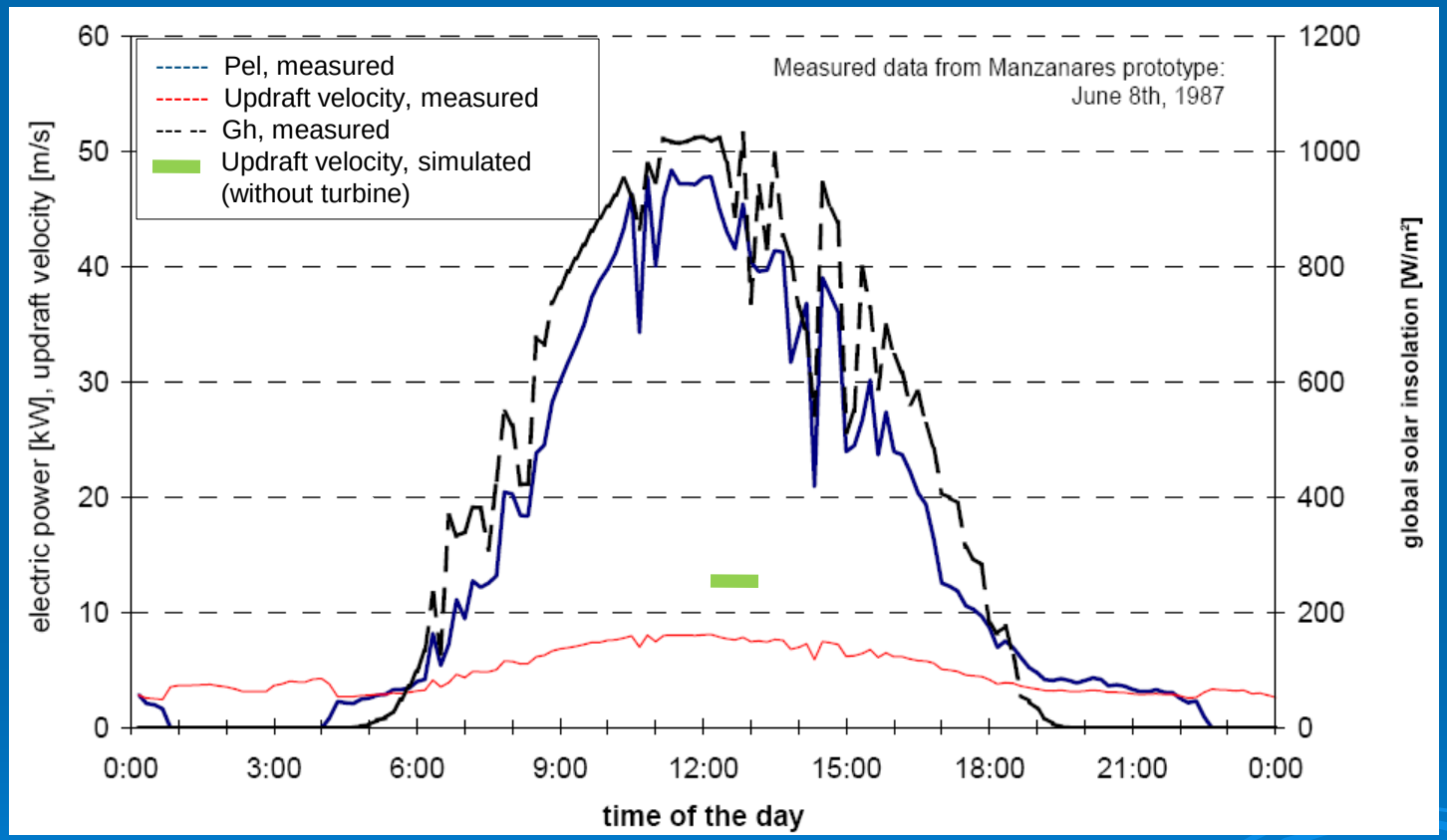


$P_0 = (0;10)$ $P_1 = (-1.6;10)$ $P_2=(-10;0)$ $P_3 = (-132;-10)$ $P_4 = (-132;10)$ $P_5 = (-122;1.85)$

$P_6 = (-13.3; 1.85)$ $P_7 = (-5; 10)$ $P_8 = (-5; 194.6)$ $P_9 = (-15; 204.6)$

4. Application example





Comparison of Numerical Simulation and Experimental* Results

*Jorg Schlaich, Rudolf Bergermann, Wolfgang Schiel, Gerhard Weinreber; Design of commercial Solar Updraft Tower Systems – Utilization of Solar Induced convective Flows for Power Generation

Optimization Procedure Layout

```
for Geometric parameter 1 = di:step1:d1f
    for  Geometric parameter 2 = d2i:step2:d2f
            .
              .
                .
            for Geometric parameter n = dni: stepn: dnf
              - Solve governing equations of the analyzed system
              - Evaluate optimization variable(s).
              - Save optimum variables values(s) and corresponding
                geometric parameters.
            end
              .
             .
            .
    end
end
```

6. Concluding Remarks

- Main considerations for modeling a wide number of energy systems have been presented. Although the 3D, time dependent, model has a high computational cost, it has the main advantage of giving more realistic solutions.

- The numerical approach presented can also be used to model highly conducting fluids in presence of a electromagnetic (EM) field, in which case the inclusion of the governing equations of EM fields have to be included.

- Analytical solutions of the governing equations of fluid flows for a system with a generalized 3D geometry, as presented, have not been obtained yet, and only solutions by numerical methods are feasible now.

- Ways to lower the computational time will be explored on future applications of the model presented for energy systems.