SOLAR 2010:

THREE DIMENSIONAL MATHEMATICAL MODEL OF TIME–DEPENDENT CONVECTION OF FLUIDS IN ENERGY SYSTEMS THAT USE SOLAR RADIATION

Rafael O. Castro, John Duffy

U Mass Lowell
OUTLINE

1. Introduction
2. Modeling of Convection of fluids
3. Numerical Method
4. Application Example
5. Design Optimization
6. Concluding Remarks
1. Introduction

World Energy Demand

World CO₂ Emissions

Objective:

To present an overview of the main aspects for the mathematical modeling of convection of fluids that are present in a wide number of energy systems.
2. Modeling of Convection

The mathematical modeling process:

- Laws of physics
- Model set-up
- Solution and verification
- Validation

source: nasa
2. Modeling of Convection

What are the governing equations of fluid flows?
Governing equations:

Conservation of mass

\[ \frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{V}) = 0 \]

Conservation of momentum

\[ \frac{\partial \rho \vec{V}}{\partial t} + \rho \vec{V} \cdot \nabla \vec{V} + \nabla p = - \nabla \Phi + \frac{1}{\rho} \left( \frac{\partial \rho}{\partial t} \right) \vec{V} \]

Conservation of energy

\[ \rho C_p \frac{\partial T}{\partial t} + \rho C_p \vec{V} \cdot \nabla T = \nabla \cdot ( - \kappa \nabla T + q_r) + \left( \frac{\partial \rho \rho}{\partial \ln T} \right) \frac{Dp}{Dt} \]

Diffusive fluxes

\[ \vec{t} = -\mu (\nabla \vec{V} + (\nabla \vec{V})^\dagger) + \left( \frac{2}{3} \mu - \kappa \right) \nabla (\nabla \cdot \vec{V}) \]

\[ q_r = -K \nabla T + q_r' \]
Turbulent flows are always dissipative. Viscous shear stresses perform deformation work which increases the internal energy of the fluid at the expense of kinetic energy of turbulence.

**K-epsilon:**

Equation for the turbulent kinetic energy $\kappa$:

$$\frac{\partial}{\partial t} (\rho \kappa) + \frac{\partial}{\partial x_i} (\rho \kappa u_i) = \frac{\partial}{\partial x_j} \left[ \left( \mu + \frac{\mu_t}{\sigma_\kappa} \right) \frac{\partial \kappa}{\partial x_j} \right] - \rho \varepsilon + \tau_{ij} \frac{\partial u_i}{\partial x_j},$$

Equation for the turbulent kinetic energy dissipation rate, $\varepsilon$:

$$\frac{\partial (\rho \varepsilon)}{\partial t} + \frac{\partial (\rho \varepsilon u_i)}{\partial x_i} = \frac{\partial}{\partial x_j} \left[ \left( \mu + \frac{\mu_t}{\sigma_\varepsilon} \right) \frac{\partial \varepsilon}{\partial x_j} \right] + C_{1\varepsilon} \frac{\varepsilon}{\kappa} \tau_{ij} \frac{\partial u_i}{\partial x_j} - C_{2\varepsilon} \rho \frac{\varepsilon^2}{\kappa}.$$

$$\mu_t = \rho C_{\mu} \frac{\kappa^2}{\varepsilon}$$
Three dimensional representation of energy systems that use solar radiation
Solar radiation:
Thermodynamic and Transport Properties

States of matter

- solid
- liquid
- gas
- plasma

COLD

HOT

Specific heat:
- Specific heat at constant pressure
  \[ c_p = \frac{T}{N} \left( \frac{\partial S}{\partial T} \right)_p = -\frac{T}{N} \frac{\partial^2 G}{\partial T^2} \]
- Specific heat at constant volume
  \[ c_v = \frac{T}{N} \left( \frac{\partial S}{\partial T} \right)_v = -\frac{T}{N} \frac{\partial^2 A}{\partial T^2} \]

Coefficient of thermal expansion:

\[ \alpha = \frac{1}{V} \left( \frac{\partial V}{\partial T} \right)_p = \frac{1}{V} \frac{\partial^2 G}{\partial P \partial T} \]

Compressibility:

- Isothermal compressibility
  \[ \beta_T = -\frac{1}{V} \left( \frac{\partial V}{\partial P} \right)_T = -\frac{1}{V} \frac{\partial^2 G}{\partial P^2} \]
- Adiabatic compressibility
  \[ \beta_S = -\frac{1}{V} \left( \frac{\partial V}{\partial P} \right)_S = -\frac{1}{V} \frac{\partial^2 H}{\partial P^2} \]

\[ R_g = K_g \frac{\sum z n_z}{\sum z n_z m_z} \quad \partial \rho = \rho_p \partial p + \rho_T \partial T \quad \rho = \frac{p}{R_g T} \]
3. Numerical Method

Solver Layout

**Loop: Time stepping**

\[ \text{Res}(t, X, Y, \dot{Y}) = 0 \rightarrow \text{Res}(Y) = 0 \]

- Second order implicit predictor - multicorrector
  (Jansen, 2000)

**Loop: Solution Non-linear System**

- Globalized Newton-Krylov method
  (Kelly, 2003)

\[ \left\| \text{Res} + \text{Jac}\Delta Y^{k+1} \right\| \leq \eta \left\| \text{Res} \right\| \]

\[ Y^{k+1} = Y^k + \lambda \Delta Y^{k+1} \]

**Loop: Solution Linear System**

- Preconditioned Generalized Minimal residual (GMRS)
  (Scaling, Pre-preconditioning, Preconditioning)

\[ Ax = b \]

\[ (P^{-1}A)x = (P^{-1}b) \]
4. Application example

Solar Updraft Tower
4. Application example

\[ P_0 = (0; 10) \quad P_1 = (-1.6; 10) \quad P_2 = (-10; 0) \quad P_3 = (-132; -10) \quad P_4 = (-132; 10) \quad P_5 = (-122; 1.85) \]

\[ P_6 = (-13.3; 1.85) \quad P_7 = (-5; 10) \quad P_8 = (-5; 194.6) \quad P_9 = (-15; 204.6) \]
4. Application example
Comparison of Numerical Simulation and Experimental* Results

* Jorg Schlaich, Rudolf Bergermann, Wolfgang Schiel, Gerhard Weinreber; Design of commercial Solar Updraft Tower Systems – Utilization of Solar Induced convective Flows for Power Generation
for Geometric parameter 1 = di:step1:d1f
  for Geometric parameter 2 = d2i:step2:d2f
    .
    .
    .
    for Geometric parameter n = dni:stepn:dnf
      - Solve governing equations of the analyzed system
      - Evaluate optimization variable(s).
      - Save optimum variables values(s) and corresponding geometric parameters.
    end
  end
end
- Main considerations for modeling a wide number of energy systems have been presented. Although the 3D, time dependent, model has a high computational cost, it has the main advantage of giving more realistic solutions.

- The numerical approach presented can also be used to model highly conducting fluids in presence of a electromagnetic (EM) field, in which case the inclusion of the governing equations of EM fields have to be included.

- Analytical solutions of the governing equations of fluid flows for a system with a generalized 3D geometry, as presented, have not been obtained yet, and only solutions by numerical methods are feasible now.
- Ways to lower the computational time will be explored on future applications of the model presented for energy systems.